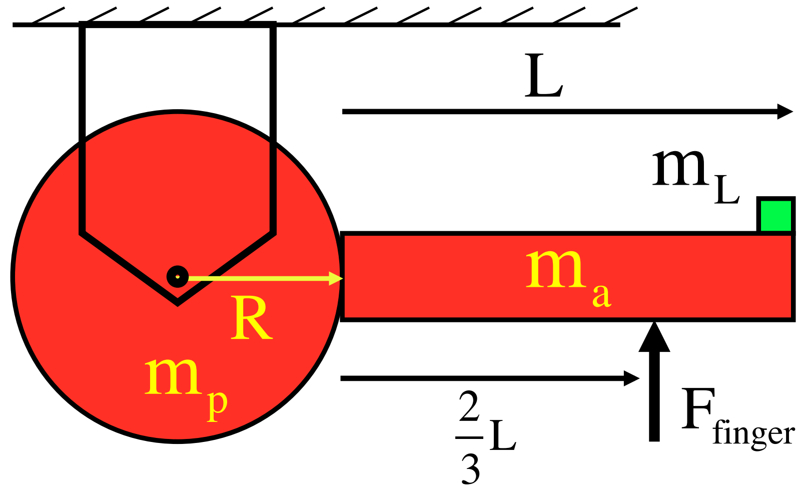
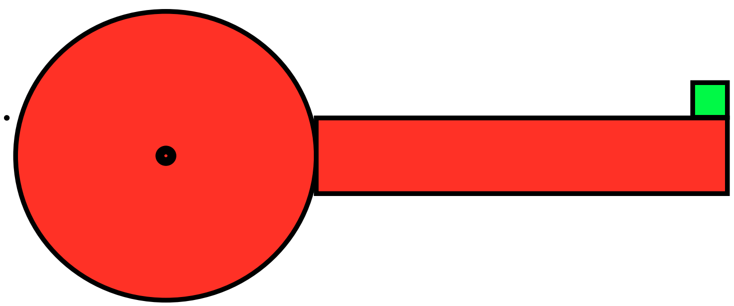
6.) An arm of length "L" is welded to a pulley of radius R. The system is pinned at the pulley's center of mass. A lump is glued to the pulley/arm's end. The system is initially held in place by a finger force F. Known:



$$m_L,\ m_a,\ m_p, R, g, L, I_{\text{arm's c. of m.}} = \frac{1}{12} m_a L^2,$$

$$F \text{ and } I_{cm,pully} = \frac{1}{2} m_p R^2$$

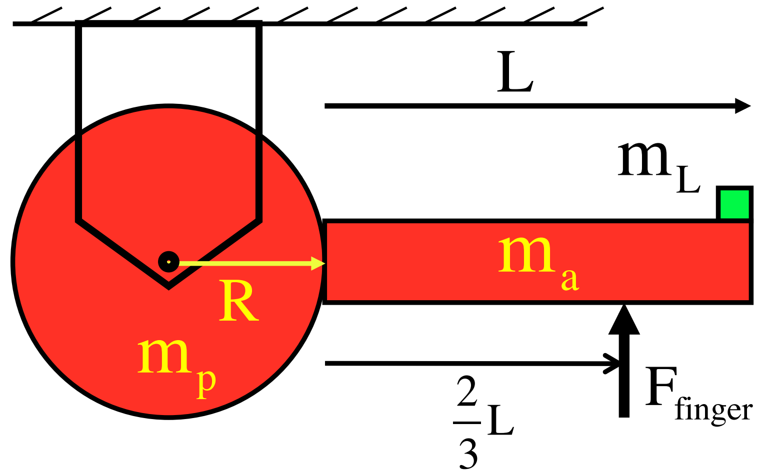
a.) Draw a f.b.d. for the forces acting on the pulley/arm system. (Hint: Note that as there is only a vertical force initially acting at the pin, and there are 5 of these forces).



b.) Use the *parallel axis theorem* to determine the ARM' S *moment of inertia* about the pin in terms of R.

$$m_L,\ m_a,\ m_p,\ R,\ g,\ L,\ I_{\text{arm's c. of m.}} = \frac{1}{12} m_a L^2$$

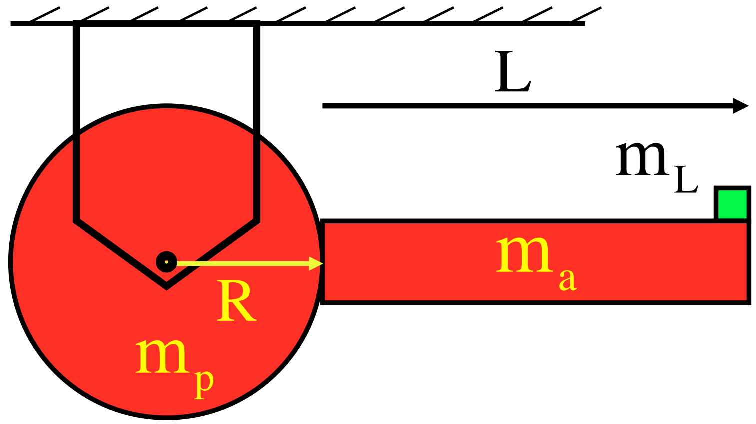
$$F \text{ and } I_{\text{cm,pully}} = \frac{1}{2} m_p R^2$$



c.) Determine the finger force required to keep the system in equilibrium.

d.) Determine the LUMP' S *moment of inertia* about the pin (the pulley' s center). Then, determine the total *moment of inertia* for ALL of the masses in the system about the pin.

$$m_L,\ m_a,\ m_p,\ R,\ g,\ L,\ I_{\text{arm's c. of m.}} = \frac{1}{12} m_a L^2$$
$$F \text{ and } I_{cm,pully} = \frac{1}{2} m_p R^2$$

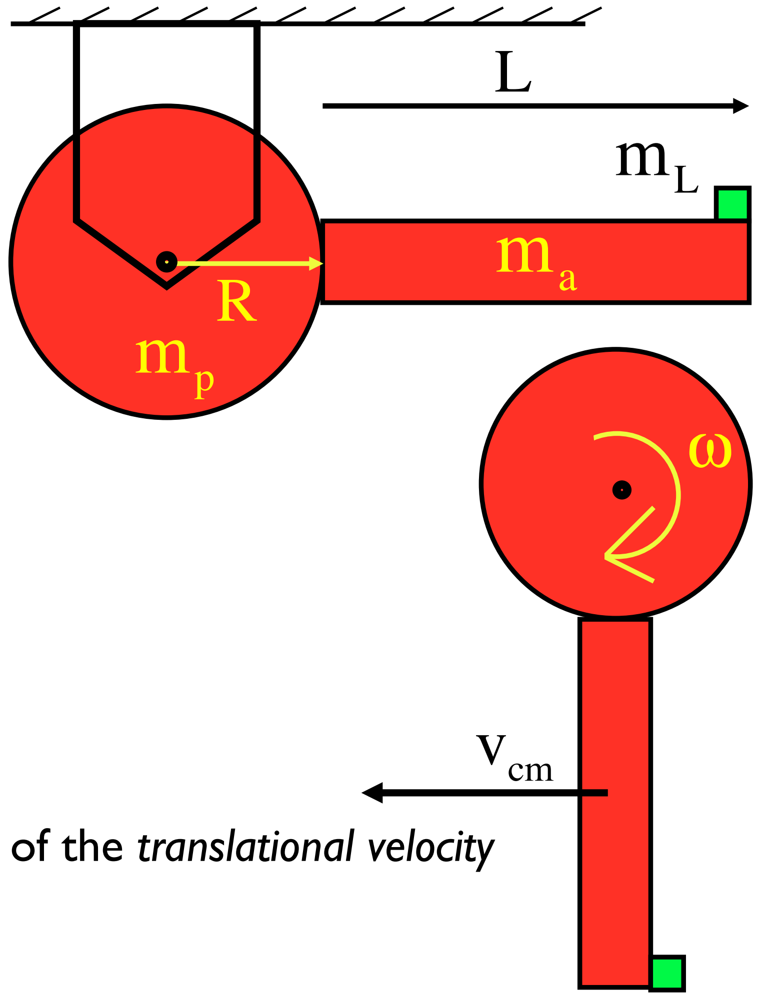


e.) With the finger removed and the system released, what is the initial *angular acceleration* of the pulley/arm?

f.) What is the initial magnitude of the acceleration of the lump?

$$m_L,\ m_a,\ m_p,\ R,\ g,\ L,\ I_{\text{arm's c. of m.}} = \frac{1}{12}m_aL^2$$
$$F \text{ and } I_{cm,pully} = \frac{1}{2}m_pR^2$$

g.) The entire system rotates down with the the arm passing through the vertical. At that point, what is the system's *angular velocity*?



h.) At the point alluded to in #g, what is the magnitude of the *translational velocity* of the arm's *center of mass*?

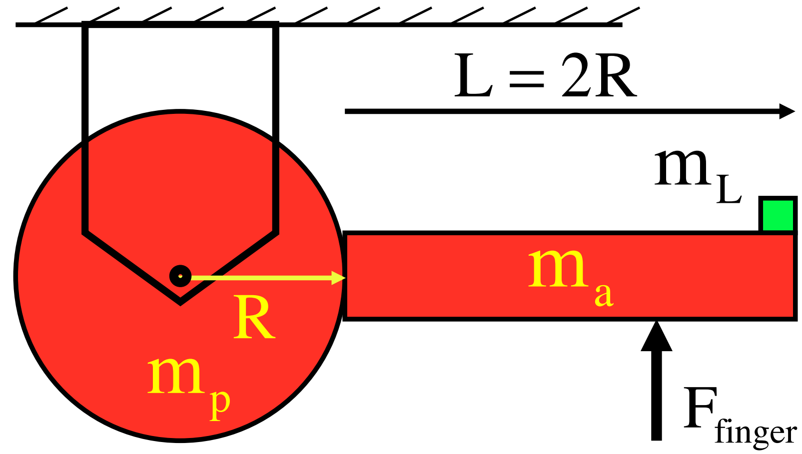
i.) At the point alluded to in #g, what is the *angular momentum* of the system about the pin?

SOLUTIONS:

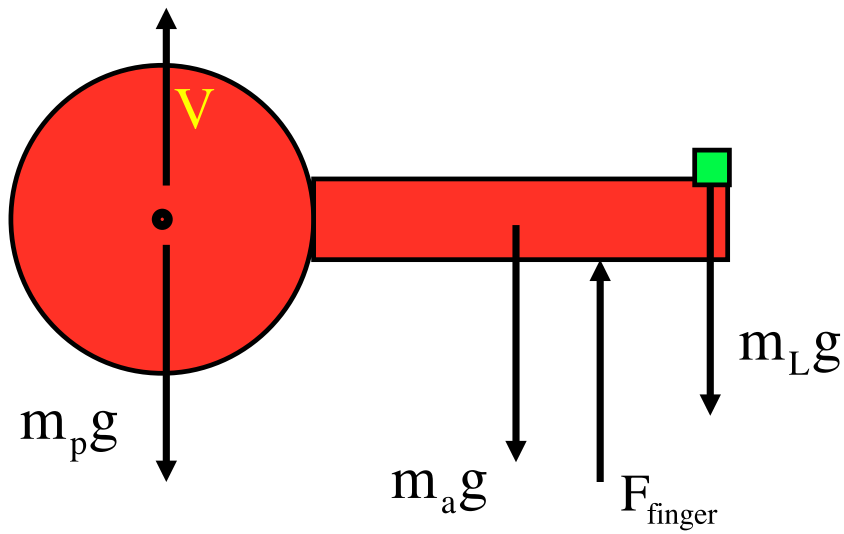
6.) An arm of length "L" is welded to a pulley of radius R pinned at the pulley's center of mass. A lump is glued to the pulley/arm's end. The system is initially held in place by a finger force F. Known:

$$m_L,\ m_a,\ m_p,\ R,\ g,\ L{=}2R,\ I_{\text{arm's c. of m.}} = \frac{1}{3}m_aR^2$$

$$F \text{ and } I_{cm,pully} = \frac{1}{2}m_pR^2$$



a.) Draw a f.b.d. for the forces acting on the pulley/arm system. (Hint: Note that as there is only a vertical force initially acting at the pin, and there are 5 of these forces).



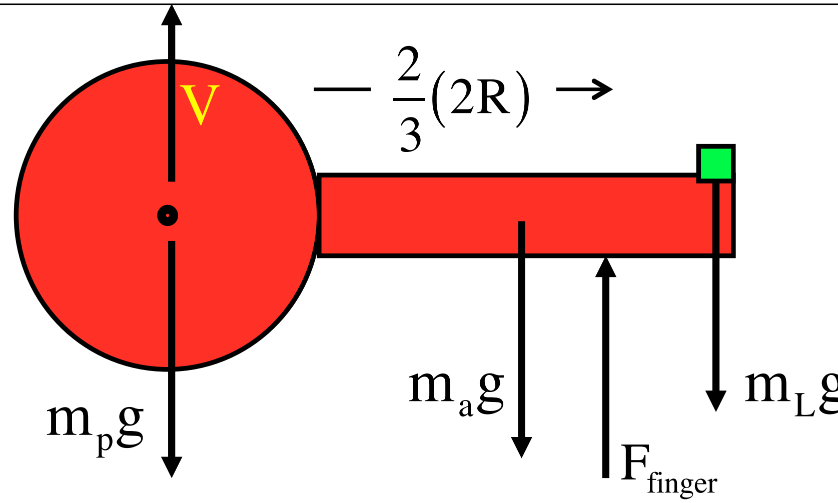
b.) Use the *parallel axis theorem* to determine the ARM'S *moment of inertia* about the pin in terms of R.

$$I_P = I_{cm} + md^2$$

$$= \left(\frac{1}{3}m_aR^2\right) + m_a(2R)^2$$

$$= \frac{13}{3}m_aR^2$$

c.) Determine the finger force required to keep the system in equilibrium.



As usual, this is most easily done summing torques, in this case about the pulley's *center of mass* (that eliminates the V and pulley-mass force), and putting that sum equal to zero (as alpha is zero). Noting the distances, and noting that because the beam is in the horizontal, "r-perpendicular" is just "r" in all cases, we can write:

$$R + R = 2R$$

$$\left(R + \frac{2}{3}(2R)\right) = \frac{7}{3}R$$

$$\left(R + (2R)\right) = 3R$$

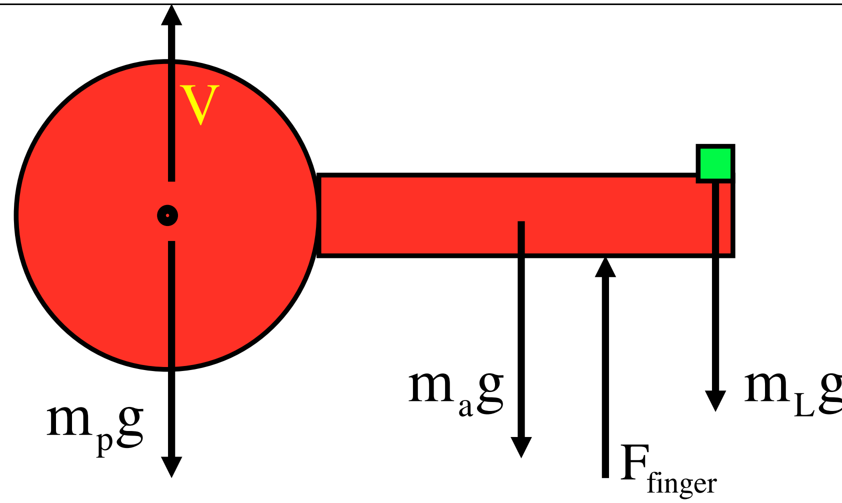
$$\sum \Gamma_{pin}:$$

$$-m_a g(2R) + f_{finger}\left(\frac{7}{3}R\right) - m_L g(3R) = I_{pin}\alpha \quad (\alpha = 0)$$

$$\Rightarrow \quad f_{finger} = \frac{2m_a g + 3m_L g}{7/3} = \frac{6m_a g + 9m_L g}{7}$$

$$= \frac{6m_a g + 9m_L g}{7}$$

d.) Determine the LUMP'S *moment of inertia* about the pin (the pulley's center). Then, determine the TOTAL *moment of inertia* for ALL of the masses in the system about the pin.



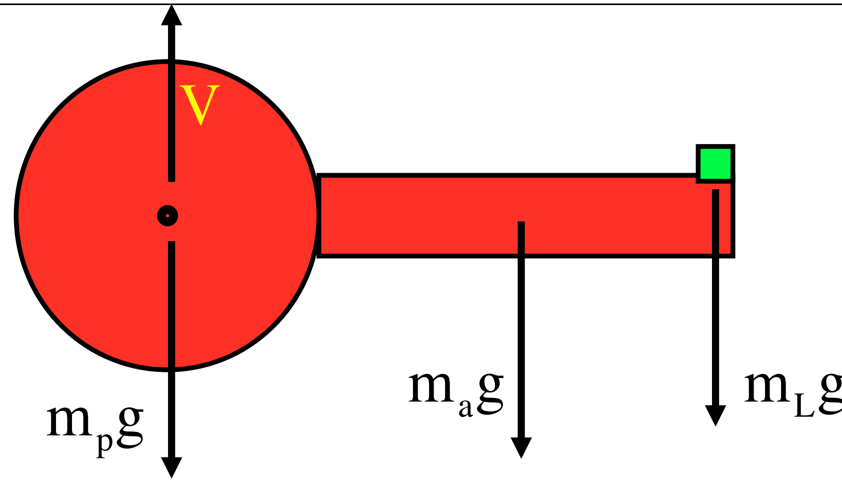
lump (treated like a point mass):

$$I_{p,lump} = m_L (3R)^2$$
$$= 9m_L R^2$$

NET *moment of inertia* about the pin due all of the mass:

$$I_{pin,tot} = I_{p,lump} + I_{pulley} + I_{p,arm}$$
$$= 9m_L R^2 + \frac{1}{2}m_p R^2 + \frac{13}{3}m_a R^2$$
$$= \left(9m_L + \frac{1}{2}m_p + \frac{13}{3}m_a\right)R^2$$

e.) With the finger removed and the system released, what is the initial *angular acceleration* of the pulley/arm?



This is a "torque about the pin" problem:

$$\sum \Gamma_{pin}:$$

$$-m_a g(2R) - m_L g(3R) = -I_{pin}\alpha$$

$$m_a g\left(R+\frac{L}{2}\right) + m_L g(R+L) = \left[9m_L R^2 + \frac{1}{2}m_p R^2 + \frac{13}{3}m_a R^2\right]\alpha$$

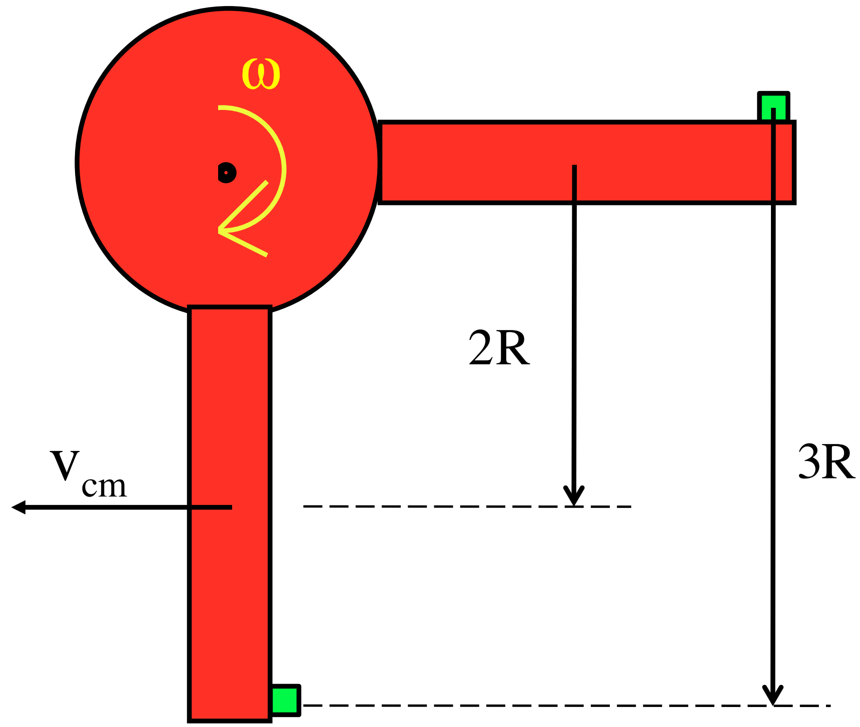
$$\Rightarrow \quad \alpha = \frac{2m_a g\not{R} + 3m_L g\not{R}}{\left(9m_L R^{\not{2}} + \frac{1}{2}m_p R^{\not{2}} + \frac{13}{3}m_a R^{\not{2}}\right)}$$

$$\Rightarrow \quad \alpha = \frac{2m_a g + 3m_L g}{\left(9m_L R + \frac{1}{2}m_p R + \frac{13}{3}m_a R\right)}$$

f.) What is the initial magnitude of the acceleration of the LUMP?

$$a_{lump} = (3R)\alpha$$

g.) The entire system rotates down with the the arm passing through the vertical. At that point, what is the system's *angular velocity*?



(Because the expression is SO LARGE, I've had to write out the *conservation of energy* relationship fairly tiny. Apologies for that.)

With the drop distance of the arm's *center of mass* identified as 2R (see sketch) and the LUMP'S drop-distance as 3R, the potential energies can be expressed in those variables and we can write:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + \left[m_b g(2R) + m_{lump} g(3R)\right] + 0 = \left[KE_{lump} + KE_{arm} + KE_{pulley}\right] + 0$$

$$= \left[\left(\frac{1}{2} m_{lump} v_{lump}^2\right) + \left(\frac{1}{2} I_{beam,pin}\,\omega^2\right) + \left(\frac{1}{2} I_{pulley}\,\omega^2\right)\right] + 0$$

$$= \left[\left(\frac{1}{2} m_{lump}\left((3R)\omega\right)^2\right) + \left(\frac{1}{2}\left(\frac{13}{3} m_a R^2\right)\omega^2\right) + \left(\frac{1}{2}\left(\frac{1}{2} m_p R^2\right)\omega^2\right)\right]$$

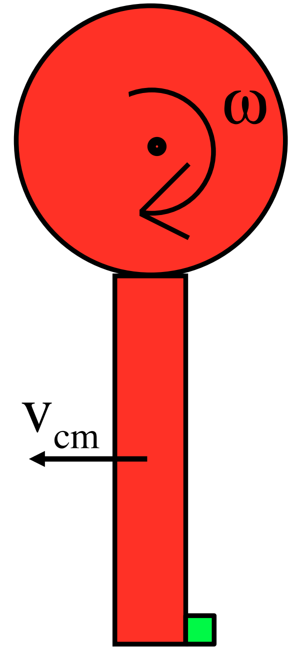
(treated like point mass)

("I" from Parallel Axis Theorem)

g.) (con't.) Solving yields:

$$\omega = \sqrt{\frac{2m_b g\cancel{R} + 3m_{lump} g\cancel{R}}{\frac{3}{2}m_{lump}R^{\cancel{2}} + \frac{1}{2}\left(\frac{13}{3}m_a R^{\cancel{2}}\right) + \frac{1}{2}\left(\frac{1}{2}m_p R^{\cancel{2}}\right)}}$$

$$= \sqrt{\frac{4m_b g + 6m_{lump} g}{\left(3m_{lump} + \frac{13}{3}m_a + \frac{1}{2}m_p\right)R}}$$



h.) At the point alluded to in #g, what is the magnitude of the *translational velocity* of the arm's *center of mass*?

$$v_{rod,cm} = (2R)\omega$$

i.) At the point alluded to in #g, what is the *angular momentum* of the system about the pin?

$$L = I_{total}\omega$$